Kurs Komputerowy S
System Symboliczny
Mathematica

Obliczenia symboliczne

■ Definiowanie zmiennych

zmienna = wartosc
Set[zmienna,wartosc]

In[1]:=
```
a1 = 5
```
Out[1]=
```
5
```
In[2]:=
```
b1 = a1^4
```
Out[2]=
```
625
```
In[3]:=
```
a1 = 15
```
Out[3]=
```
15
```
In[4]:=
```
b1
```
Out[4]=
```
625
```

In[5]:=
```
b2 = a2^4
```
Out[5]=
```
a2^4
```

In[6]:=
```
a2 = 5
b2
```
Out[6]=
```
5
```

Out[7]=
```
625
```

In[8]:=
```
a2 = 15
b2
```
Out[8]=
```
15
```

Out[9]=
```
50 625
```

In[10]:=
```
a1 + b1
```
Out[10]=
```
640
```

In[11]:=
```
Set[c1, a1^3]
```
Out[11]=
```
3375
```

In[12]:=
```
c1
```
Out[12]=
```
3375
```

In[13]:=
```
Clear[a2, b2]
```

In[14]:=
```
a2
```
Out[14]=
```
a2
```

In[15]:=
```
b2 = a2^4
```
Out[15]=
```
a2^4
```

In[16]:=
```
a2 = 5
```
Out[16]=
```
5
```
In[17]:=
```
b2
```
Out[17]=
```
625
```
In[18]:=
```
a2 = 15
```
Out[18]=
```
15
```
In[19]:=
```
b2
```
Out[19]=
```
50 625
```

zmienna := wartosc
SetDelayed[zmienna,wartosc]

In[20]:=
```
a3 = 5
```
Out[20]=
```
5
```
In[21]:=
```
b3 := a3^4
```
In[22]:=
```
b3
```
Out[22]=
```
625
```
In[23]:=
```
a3 = 15
```
Out[23]=
```
15
```
In[24]:=
```
b3
```
Out[24]=
```
50 625
```

zmienna = .
Unset[zmienna]

In[25]:=
```
Quit
```

In[1]:=
```
a = 5
b = 34
c = 8
```

Out[1]=
```
5
```

Out[2]=
```
34
```

Out[3]=
```
8
```

In[4]:=
```
a = .
```

In[5]:=
```
a + b
```

Out[5]=
```
34 + a
```

In[6]:=
```
? a
```

```
Global`a
```

In[7]:=
```
Clear[a]
```

In[8]:=
```
? a
```

```
Global`a
```

In[9]:=
```
Unset[c]
```

In[10]:=
```
c
```

Out[10]=
```
c
```

Podstawianie zmiennych:

```
/.   (*  ReplaceAll[]  *)
//.  (* ReplaceRepeated[]   *)
```

In[11]:=
```
b2 = a2^4
```

Out[11]=
$$a2^4$$

In[12]:=
```
b2 /. a2 -> 5
```

Out[12]=
```
625
```

In[13]:=
```
a2
```

Out[13]=
```
a2
```

In[14]:=
```
a = 2 x + 7
```

Out[14]=
$$7 + 2\,x$$

In[15]:=
```
x = 2
a
```

Out[15]=
```
2
```

Out[16]=
```
11
```

In[17]:=
```
x = 3
a
```

Out[17]=
```
3
```

Out[18]=
```
13
```

In[19]:=
```
x
```

Out[19]=
```
3
```

In[20]:=
```
x =.
```

In[21]:=
```
x
```
Out[21]=
```
x
```

In[22]:=
```
a /. x -> 2
a /. x -> 3
```
Out[22]=
```
11
```
Out[23]=
```
13
```

In[24]:=
```
x
```
Out[24]=
```
x
```

In[25]:=
```
a =.
b =.
```

In[27]:=
```
x^2 + a x /. x -> a
```
Out[27]=
$2\,a^2$

In[28]:=
```
x^2 + a x //. {x -> a, a -> b}
```
Out[28]=
$2\,b^2$

In[29]:=
```
x^2 + 4 x //. {x -> a, a -> x}
```
ReplaceRepeated::rrlim : Exiting after $4\,x + x^2$ scanned 65536 times. ≫

Out[29]=
$4\,x + x^2$

■ Funkcje i wielomiany

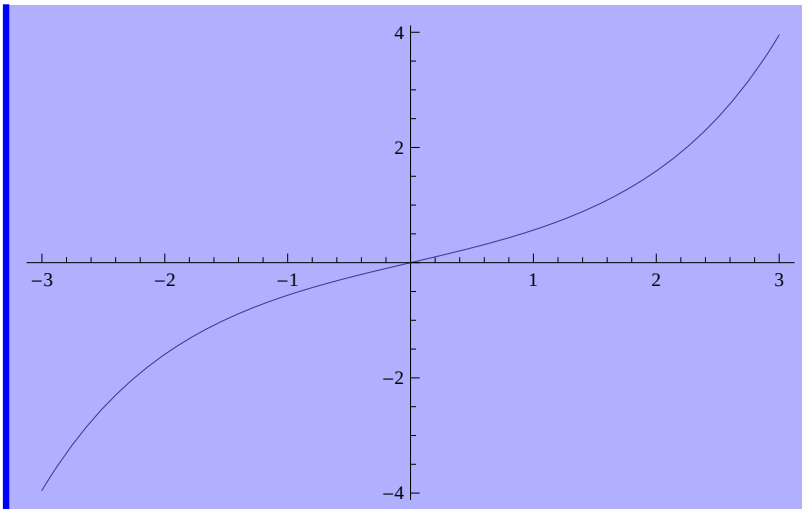
BesselI[n, z]
BesselJ[n,z]
BesselK[n,z]
BesselY[n,z]
BernoulliB[n,x]
...

In[30]:=
```
BesselI[0, 1.]
```

Out[30]=
```
1.26607
```

In[31]:=
```
Plot[BesselI[1, x], {x, -3, 3}]
```

Out[31]=

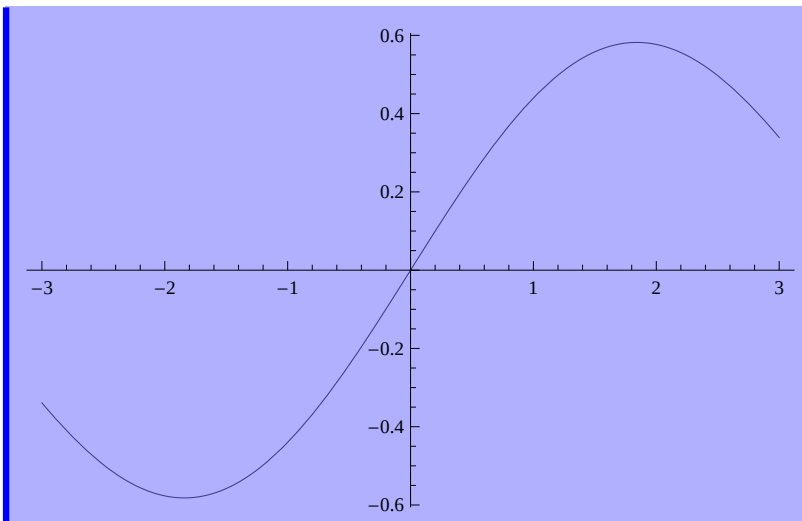


In[32]:=
```
BesselI[1, 2.0]
```

Out[32]=
```
1.59064
```

In[33]:=
```
Plot[BesselJ[1, x], {x, -3, 3}]
```

Out[33]=



In[34]:=
```
BernoulliB[2, x]
```

Out[34]=
$$\frac{1}{6} - x + x^2$$

ChebyshevT[n, x]
ChebyshevU[n,x]
HermiteH[n,x]
LaguerreL[n,a,x]
LegendreP[n,x]
...

In[35]:=
```
ChebyshevT[4, x]
```

Out[35]=
$$1 - 8\,x^2 + 8\,x^4$$

In[36]:=
```
ChebyshevU[4, x]
```

Out[36]=
$$1 - 12\,x^2 + 16\,x^4$$

In[37]:=
```
HermiteH[4, x]
```

Out[37]=
$$12 - 48\,x^2 + 16\,x^4$$

In[38]:=
```
LaguerreL[4, p, x]
```

Out[38]=
$$\frac{1}{24}\left(24 + 50\,p + 35\,p^2 + 10\,p^3 + p^4 - 96\,x - 104\,p\,x - 36\,p^2\,x - 4\,p^3\,x + 72\,x^2 + 42\,p\,x^2 + 6\,p^2\,x^2 - 16\,x^3 - 4\,p\,x^3 + x^4\right)$$

In[39]:=
```
LegendreP[4, x]
```

Out[39]=
$$\frac{1}{8}\left(3 - 30\,x^2 + 35\,x^4\right)$$

■ Dzialania na liczbach

Permutations[lista]

Binomial[n,m]
Multinomial[n1,n2,...]
FactorInteger[liczba]
GCD[l1,l2]
LCM[l1,l2]

In[40]:=
```
a = Permutations[{1, 2, 3, 4, 5, 6, 7}];
```

In[41]:=
```
a[[5]]
```

Out[41]=
```
{1, 2, 3, 4, 7, 5, 6}
```

In[42]:=
```
7!
```

Out[42]=
```
5040
```

In[43]:=
```
Binomial[4, 2]  (* n!/(m!(n-m)!) *)
```

Out[43]=
```
6
```

In[44]:=
```
Multinomial[2, 3, 4]  (* (n1+n2+...)!/(n1!n2!...) *)
```

Out[44]=
```
1260
```

In[45]:=
```
2^4 * 5^3 * 9 * 7^2
```

Out[45]=
```
882 000
```

In[46]:=
```
FactorInteger[%]
```

Out[46]=
```
{{2, 4}, {3, 2}, {5, 3}, {7, 2}}
```

In[47]:=
```
GCD[60, 45]
```

Out[47]=
```
15
```

In[48]:=
```
LCM[60, 45]
```

Out[48]=
```
180
```

Zaokraglanie liczb:

Ceiling[liczba]
Floor[liczba]
Round[liczba]

In[49]:=
```
Ceiling[2.4]
Ceiling[-2.4]
```

Out[49]=
```
3
```

Out[50]=
```
-2
```

In[51]:=
```
Floor[2.4]
Floor[-2.4]
```

Out[51]=
```
2
```

Out[52]=
```
-3
```

In[53]:=
```
Round[2.4]
Round[-2.4]
```

Out[53]=
```
2
```

Out[54]=
```
-2
```

■ Dzialania na wyrazeniach algebraicznych

Numerator[wyr]
Denominator[wyr]
ExpandNumerator[wyr]
ExpandDenominator[wyr]
Together[wyr]
Apart[wyr]

In[55]:=
```
e = (x - 1)^2 (2 + x) / ((1 + x) (x - 3)^2)
```

Out[55]=
$$\frac{(-1+x)^2\,(2+x)}{(-3+x)^2\,(1+x)}$$

In[56]:=
```
Numerator[e]
```

Out[56]=
$$(-1+x)^2\,(2+x)$$

In[57]:=
```
Denominator[e]
```

Out[57]=
$$(-3+x)^2\,(1+x)$$

In[58]:=
```
ExpandNumerator[e]
```

Out[58]=
$$\frac{2-3x+x^3}{(-3+x)^2\,(1+x)}$$

In[59]:=
```
ExpandDenominator[e]
```

Out[59]=
$$\frac{(-1+x)^2\,(2+x)}{9+3x-5x^2+x^3}$$

In[60]:=
```
e1 = (x + 2) / ((x^2 - 9))
e2 = (x - 4) / (x + 3)
```

Out[60]=
$$\frac{2+x}{-9+x^2}$$

Out[61]=
$$\frac{-4+x}{3+x}$$

In[62]:=
```
e1 + e2
```

Out[62]=
$$\frac{-4+x}{3+x}+\frac{2+x}{-9+x^2}$$

In[63]:=
```
Together[e1 + e2]
```

Out[63]=
$$\frac{14-6x+x^2}{(-3+x)\,(3+x)}$$

In[64]:=
```
ExpandDenominator[Together[e1 + e2]]
```

Out[64]=
$$\frac{14 - 6\,x + x^2}{-9 + x^2}$$

In[65]:=
```
(x^2 + 1) / (x - 1)
```

Out[65]=
$$\frac{1 + x^2}{-1 + x}$$

In[66]:=
```
Apart[(x^2 + 1) / (x - 1)]
```

Out[66]=
$$1 + \frac{2}{-1 + x} + x$$

■ Dzialania na wielomianach

Expand[wyr]
Factor[wyr]
Simplify[wyr]
FullSimplify[wyr]
ExpandAll[wyr]
PowerExpand[wyr]
TrigExpand[wyr]
ComplexExpand[wyr]

In[67]:=
```
(2 + x)^10
```

Out[67]=
$$(2 + x)^{10}$$

In[68]:=
```
Expand[%] + 1
```

Out[68]=
$$1025 + 5120\,x + 11\,520\,x^2 + 15\,360\,x^3 + 13\,440\,x^4 + 8064\,x^5 + 3360\,x^6 + 960\,x^7 + 180\,x^8 + 20\,x^9 + x^{10}$$

In[69]:=
```
Factor[%]
```

Out[69]=
$$\left(5 + 4\,x + x^2\right)\left(205 + 860\,x + 1575\,x^2 + 1640\,x^3 + 1061\,x^4 + 436\,x^5 + 111\,x^6 + 16\,x^7 + x^8\right)$$

In[70]:=
```
(x^2 + 2 x + 1) / (x + 1)
```

Out[70]=
$$\frac{1 + 2\,x + x^2}{1 + x}$$

In[71]:=
```
Simplify[%]
```

Out[71]=
$$1 + x$$

In[72]:=
```
e
```

Out[72]=
$$\frac{(-1 + x)^2\,(2 + x)}{(-3 + x)^2\,(1 + x)}$$

In[73]:=
```
Expand[e]
```

Out[73]=
$$\frac{2}{(-3 + x)^2\,(1 + x)} - \frac{3\,x}{(-3 + x)^2\,(1 + x)} + \frac{x^3}{(-3 + x)^2\,(1 + x)}$$

In[74]:=
```
Expand[%]
```

Out[74]=
$$\frac{2}{(-3 + x)^2\,(1 + x)} - \frac{3\,x}{(-3 + x)^2\,(1 + x)} + \frac{x^3}{(-3 + x)^2\,(1 + x)}$$

In[75]:=
```
ExpandAll[e]
```

Out[75]=
$$\frac{2}{9 + 3\,x - 5\,x^2 + x^3} - \frac{3\,x}{9 + 3\,x - 5\,x^2 + x^3} + \frac{x^3}{9 + 3\,x - 5\,x^2 + x^3}$$

In[76]:=
```
q = Sqrt[x y]
```

Out[76]=
$$\sqrt{x\,y}$$

In[77]:=
```
Expand[q]
```

Out[77]=
$$\sqrt{x\,y}$$

In[78]:=
```
ExpandAll[q]
```

Out[78]=
$$\sqrt{x\,y}$$

In[79]:=
```
PowerExpand[q]
```

Out[79]=
$$\sqrt{x}\,\sqrt{y}$$

In[80]:=
```
Expand[Sin[2 x]]
```

Out[80]=
```
Sin[2 x]
```

In[81]:=
```
TrigExpand[Sin[2 x]]
```

Out[81]=
```
2 Cos[x] Sin[x]
```

In[82]:=
```
Simplify[%]
```

Out[82]=
```
Sin[2 x]
```

In[83]:=
```
Expand[Sin[x + I y]]
```

Out[83]=
```
Sin[x + i y]
```

In[84]:=
```
ComplexExpand[Sin[x + I y]]
```

Out[84]=
```
Cosh[y] Sin[x] + i Cos[x] Sinh[y]
```

In[85]:=
```
Simplify[%]
```

Out[85]=
```
Sin[x + i y]
```

In[86]:=
```
? *Expand*
```

▼ System‘

ButtonExpandable	ExpandDenominator	LogicalExpand	TrigExpand
ComplexExpand	ExpandFileName	PiecewiseExpand	
Expand	ExpandNumerator	PowerExpand	
ExpandAll	FunctionExpand	TransferFunctionExpand	

Collect[wiel, zm]
Coefficient[wiel,wyr]
CoefficientList[wiel,zm]
Exponent[wiel,wyr]

In[87]:=
```
c = Expand[(2 x + 3 y + 2)^3 * 2 x (4 y - 2)^2]
```

Out[87]=
$$64\,x + 192\,x^2 + 192\,x^3 + 64\,x^4 + 32\,x\,y - 192\,x^2\,y - 480\,x^3\,y - 256\,x^4\,y - 464\,x\,y^2 - 1104\,x^2\,y^2 - 384\,x^3\,y^2 + 256\,x^4\,y^2 - 360\,x\,y^3 + 576\,x^2\,y^3 + 1152\,x^3\,y^3 + 864\,x\,y^4 + 1728\,x^2\,y^4 + 864\,x\,y^5$$

In[88]:=
```
Collect[c, x]
```

Out[88]=
$$x^4\left(64 - 256\,y + 256\,y^2\right) + x^3\left(192 - 480\,y - 384\,y^2 + 1152\,y^3\right) + x^2\left(192 - 192\,y - 1104\,y^2 + 576\,y^3 + 1728\,y^4\right) + x\left(64 + 32\,y - 464\,y^2 - 360\,y^3 + 864\,y^4 + 864\,y^5\right)$$

In[89]:=
```
Collect[c, y]
```

Out[89]=
$$64\,x + 192\,x^2 + 192\,x^3 + 64\,x^4 + \left(32\,x - 192\,x^2 - 480\,x^3 - 256\,x^4\right)y + \left(-464\,x - 1104\,x^2 - 384\,x^3 + 256\,x^4\right)y^2 + \left(-360\,x + 576\,x^2 + 1152\,x^3\right)y^3 + \left(864\,x + 1728\,x^2\right)y^4 + 864\,x\,y^5$$

In[90]:=
```
Coefficient[c, x^5]
```

Out[90]=
```
0
```

In[91]:=
```
Coefficient[c, x y^2]
```

Out[91]=
$$-464$$

In[92]:=
```
CoefficientList[c, x]
```

Out[92]=
$$\{0,\ 64 + 32\,y - 464\,y^2 - 360\,y^3 + 864\,y^4 + 864\,y^5,\ 192 - 192\,y - 1104\,y^2 + 576\,y^3 + 1728\,y^4,\ 192 - 480\,y - 384\,y^2 + 1152\,y^3,\ 64 - 256\,y + 256\,y^2\}$$

In[93]:=
```
CoefficientList[c, y]
```

Out[93]=
$$\{64\,x + 192\,x^2 + 192\,x^3 + 64\,x^4,\ 32\,x - 192\,x^2 - 480\,x^3 - 256\,x^4,\ -464\,x - 1104\,x^2 - 384\,x^3 + 256\,x^4,\ -360\,x + 576\,x^2 + 1152\,x^3,\ 864\,x + 1728\,x^2,\ 864\,x\}$$

In[94]:=
```
Exponent[c, y]
```

Out[94]=
$$5$$

In[95]:=
```
Exponent[c, y^2]
```

Out[95]=
$$\frac{5}{2}$$

PolynomialQuotient[w1,w2,zm]
PolynomialRemainder[w1,w2,zm]

In[96]:=
```
w1 = x^5 - 4 x^4 + 3 x^3 - 2 x + 4
w2 = x^2 - 5 x + 5
```

Out[96]=
$$4 - 2\,x + 3\,x^3 - 4\,x^4 + x^5$$

Out[97]=
$$5 - 5\,x + x^2$$

In[98]:=
```
pq = PolynomialQuotient[w1, w2, x]
```

Out[98]=
$$10 + 3\,x + x^2 + x^3$$

In[99]:=
```
pr = PolynomialRemainder[w1, w2, x]
```

Out[99]=
$$-46 + 33\,x$$

In[100]:=
```
pq * w2 + pr - w1
```

Out[100]=
$$-50 + 35\,x - 3\,x^3 + 4\,x^4 - x^5 + \left(5 - 5\,x + x^2\right)\left(10 + 3\,x + x^2 + x^3\right)$$

In[101]:=
```
Simplify[(pq * w2 + pr) - w1]
```

Out[101]=
```
0
```

■ Operatory logiczne

Porownywanie:
== (* Equal[] *)
< (* Less[] *)
<= (* LessEqual[] *)
> (* Greater[] *)
>= (* GreaterEqual[] *)
!= (* Unequal[] *)

In[102]:=
```
2 == 4
```

Out[102]=
```
False
```

In[103]:=
```
3 < 5
```

Out[103]=
```
True
```

In[104]:=
```
Unequal[4, 5]
```

Out[104]=
```
True
```

In[105]:=
```
y == 4
```

Out[105]=
$$y == 4$$

```
&&    (* And[]  *)
 ||      (*  Or[]  *)
```

In[106]:=
```
3 == 5 && 5 > 4
```

Out[106]=
```
False
```

In[107]:=
```
3 == 5 || 5 > 4
```

Out[107]=
```
True
```

■ Rozwiazywanie rownan

```
Solve[rownanie, zmienna]
Reduce[rownanie,zmienna]
```

In[108]:=
```
x =.
```

In[109]:=
```
Solve[x == 4, x]
```

Out[109]=
```
{{x → 4}}
```

In[110]:=
```
Solve[x^2 + 5 x - 14 == 0, x]
```

Out[110]=
```
{{x → -7}, {x → 2}}
```

In[111]:=
```
eq = x^2 + 6 x + 5 == 0
Solve[eq, x]
```

Out[111]=
$$5 + 6\,x + x^2 == 0$$

Out[112]=
```
{{x → -5}, {x → -1}}
```

In[113]:=
```
eqp = x^2 + 6 p x + 5 == 0
```

Out[113]=
$$5 + 6\,p\,x + x^2 == 0$$

In[114]:=
```
s = Solve[eqp, x]
```

Out[114]=
$$\left\{\left\{x \to -3\,p - \sqrt{-5 + 9\,p^2}\right\}, \left\{x \to -3\,p + \sqrt{-5 + 9\,p^2}\right\}\right\}$$

In[115]:=
```
s[[1]]
```

Out[115]=
$$\left\{x \to -3\,p - \sqrt{-5 + 9\,p^2}\right\}$$

In[116]:=
```
s[[1, 1]]
```

Out[116]=
$$x \to -3\,p - \sqrt{-5 + 9\,p^2}$$

In[117]:=
```
s[[1, 1, 2]]
```

Out[117]=
$$-3\,p - \sqrt{-5 + 9\,p^2}$$

In[118]:=
```
s1 = Solve[s[[1, 1, 2]] == s[[2, 1, 2]], p]
```

Out[118]=
$$\left\{\left\{p \to -\frac{\sqrt{5}}{3}\right\}, \left\{p \to \frac{\sqrt{5}}{3}\right\}\right\}$$

In[119]:=
```
e1 = eqp /. s1
```

Out[119]=
$$\left\{5 - 2\sqrt{5}\,x + x^2 == 0,\ 5 + 2\sqrt{5}\,x + x^2 == 0\right\}$$

In[120]:=
```
Solve[e1[[1]], x]
```

Out[120]=
$$\left\{\left\{x \to \sqrt{5}\right\}, \left\{x \to \sqrt{5}\right\}\right\}$$

In[121]:=
```
Solve[e1[[2]], x]
```

Out[121]=
$$\left\{\left\{x \to -\sqrt{5}\right\}, \left\{x \to -\sqrt{5}\right\}\right\}$$

In[122]:=
```
a =.
```

In[123]:=
```
Solve[a x + b == 0, x]
```

Out[123]=
$$\left\{\left\{x \to -\frac{b}{a}\right\}\right\}$$

In[124]:=
```
Reduce[a x + b == 0, x]
```

Out[124]=
$$(b == 0 \&\& a == 0) \,||\, \left(a \neq 0 \&\& x == -\frac{b}{a}\right)$$

In[125]:=
```
Solve[Sin[x] == 1 / 2, x]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Out[125]=
$$\left\{\left\{x \to \frac{\pi}{6}\right\}\right\}$$

In[126]:=
```
Reduce[Sin[x] == 1 / 2, x]
```

Out[126]=
$$C[1] \in \text{Integers} \&\& \left(x == \frac{\pi}{6} + 2\pi C[1] \,||\, x == \frac{5\pi}{6} + 2\pi C[1]\right)$$

Solve[{rownanie1, rownanie2,...},{zmienna1, zmienna2,...}]
Reduce[{rownanie1, rownanie2,...},{zmienna1, zmienna2,...}]

$$\begin{cases} 2x + 5y - 8 = 0 \\ 4x - 3y = 12 \end{cases}$$

In[127]:=
```
Solve[{2 x + 5 y - 8 == 0, 4 x - 3 y == 12}, {x, y}]
```

Out[127]=
$$\left\{\left\{x \to \frac{42}{13},\ y \to \frac{4}{13}\right\}\right\}$$

In[128]:=
```
Reduce[{2 x + 5 y - 8 == 0, 4 x - 3 y == 12}, {x, y}]
```

Out[128]=
$$x == \frac{42}{13} \&\& y == \frac{4}{13}$$

$$\begin{cases} 2x^2 + 5y - 8 = 0 \\ 4x - 3y = 12 \end{cases}$$

In[129]:=
```
Solve[{2 x^2 + 5 y - 8 == 0, 4 x - 3 y == 12}, {x, y}]
```

Out[129]=
$$\left\{\left\{x \to \frac{1}{3}\left(-5-\sqrt{151}\right),\ y \to \frac{4}{9}\left(-14-\sqrt{151}\right)\right\},\ \left\{x \to \frac{1}{3}\left(-5+\sqrt{151}\right),\ y \to \frac{4}{9}\left(-14+\sqrt{151}\right)\right\}\right\}$$

In[130]:=
```
Reduce[{2 x^2 + 5 y - 8 == 0, 4 x - 3 y == 12}, {x, y}]
```

Out[130]=
$$\left(x == \frac{1}{3}\left(-5-\sqrt{151}\right) \,||\, x == \frac{1}{3}\left(-5+\sqrt{151}\right)\right) \&\& y == \frac{4}{3}(-3+x)$$

Eliminate[{rownanie1, rownanie2, ...}, zmienna]
Roots[rownanie_wielomianowe,zmienna]

In[131]:=
```
s = Eliminate[{2 x + 5 y - 4 z == 6, 5 x - 2 y + 4 z == 12, x + y - z == 6}, z]
```

Out[131]=
$$2x == 18 + y \&\& 13y == -90$$

In[132]:=
```
Roots[x^6 - 6 x^5 + 4 x^3 + x^2 - 5 == 0, x] // N
```

Out[132]=
$$x == -1.03527 \,||\, x == 5.8801 \,||\, x == -0.359762 - 0.795507\,i \,||\, x == -0.359762 + 0.795507\,i \,||\, x == 0.937344 - 0.445999\,i \,||\, x == 0.937344 + 0.445999\,i$$

■ Granice i ciagi

Limit[funkcja, zmienna -> wartosc]
Sum[wyrazenie,{zmienna, w_pocz,w_kon}]
Product[wyrazenie,{zmienna,w_pocz,w_kon}]

In[133]:=
```
Limit[x^2 + 1, x -> 1]
```

Out[133]=
$$2$$

In[134]:=
```
Limit[2 x^2 / (x + 2), x -> -2]
```

Out[134]=
$$\infty$$

In[135]:=
```
Limit[2 x^2 / (x + 2), x -> -2, Direction -> 1]
```

Out[135]=
$$-\infty$$

In[136]:=
```
Limit[2 x^2 / (x + 2), x -> -2, Direction -> -1]
```

Out[136]=
$$\infty$$

In[137]:=
```
Options[Limit]
```

Out[137]=
$$\{\text{Analytic} \to \text{False}, \text{Assumptions} :\to \$\text{Assumptions}, \text{Direction} \to \text{Automatic}\}$$

In[138]:=
```
Sum[1 / x^2, {x, 1.5, 10.8}]
```

Out[138]=
```
0.843956
```

In[139]:=
```
Sum[1 / x^2, {x, 1, Infinity}]
```

Out[139]=
$$\frac{\pi^2}{6}$$

In[140]:=
```
Sum[1 / x, {x, 1, 10 000}] // N
```

Out[140]=
```
9.78761
```

In[141]:=
```
Product[1 / x, {x, 1, 10}]
```

Out[141]=
$$\frac{1}{3\,628\,800}$$

In[142]:=
```
Product[1 / x, {x, 1, Infinity}]
```

Out[142]=
```
0
```

■ Rachunek rozniczkowy i calkowy

D[funkcja, zmienna]
D[funkcja,zm1,zm2,...]
D[funkcja,{zmienna,n}]

In[143]:=
```
Solve[x^2 + x + 1 == 0, x]
```

Out[143]=
$$\left\{\left\{x \to -(-1)^{1/3}\right\}, \left\{x \to (-1)^{2/3}\right\}\right\}$$

In[144]:=
```
?*Integrate*
```

▼ *System‘*

Integrate	*NIntegrate*

In[145]:=
```
D[x^2, x]
```

Out[145]=
$$2x$$

In[146]:=
```
D[x^n, x]
```

Out[146]=
$$n x^{-1+n}$$

In[147]:=
```
D[Sin[x], x]
```

Out[147]=
```
Cos[x]
```

In[148]:=
```
D[Cos[x], x]
```

Out[148]=
```
-Sin[x]
```

In[149]:=
```
D[Exp[x], x]
```

Out[149]=
$$e^x$$

In[150]:=
```
D[x^2, {x, 2}]
```

Out[150]=
$$2$$

In[151]:=
```
D[x^2, x, x]
```

Out[151]=
$$2$$

In[152]:=
```
f = x^2 + y^2 - 5 x y
D[f, x]
D[f, y]
```

Out[152]=
$$x^2 - 5\,x\,y + y^2$$

Out[153]=
$$2\,x - 5\,y$$

Out[154]=
$$-5\,x + 2\,y$$

In[155]:=
```
D[f, x, y]
D[f, y, x]
```

Out[155]=
$$-5$$

Out[156]=
$$-5$$

In[157]:=
```
y =.
```

In[158]:=
```
Dt[2 x^2 z + 5 y x, x]
```

Out[158]=
$$5\,y + 4\,x\,z + 5\,x\,\mathrm{Dt}[y, x] + 2\,x^2\,\mathrm{Dt}[z, x]$$

Integrate[funkcja, zmienna]
Integrate[funkcja,{zmienna,w_pocz,w_kon}]

In[159]:=
```
f = 2 x + 10
ff = D[f, x]
```

Out[159]=
```
10 + 2 x
```

Out[160]=
```
2
```

In[161]:=
```
Integrate[ff, x]
```

Out[161]=
```
2 x
```

In[162]:=
```
Integrate[ff, {x, 2, 5}]
```

Out[162]=
```
6
```

In[163]:=
```
∫Sin[x] dx
```

Out[163]=
```
-Cos[x]
```

In[164]:=
```
Integrate[Cos[x], x]
```

Out[164]=
```
Sin[x]
```

In[165]:=
```
Integrate[Sin[x], {x, -Pi / 2, 0}]
```

Out[165]=
```
-1
```

In[166]:=
```
Sin[Pi / 2] - Sin[-Pi / 2]
```

Out[166]=
```
2
```

In[167]:=
```
Integrate[x^2 + 4 x, x]
```

Out[167]=
$$2x^2 + \frac{x^3}{3}$$

■ Transformacje

LaplaceTransform[funkcja, t, s]
FourierTransform[funkcja,t,ω]
ZTransform[funkcja,n,z]
...

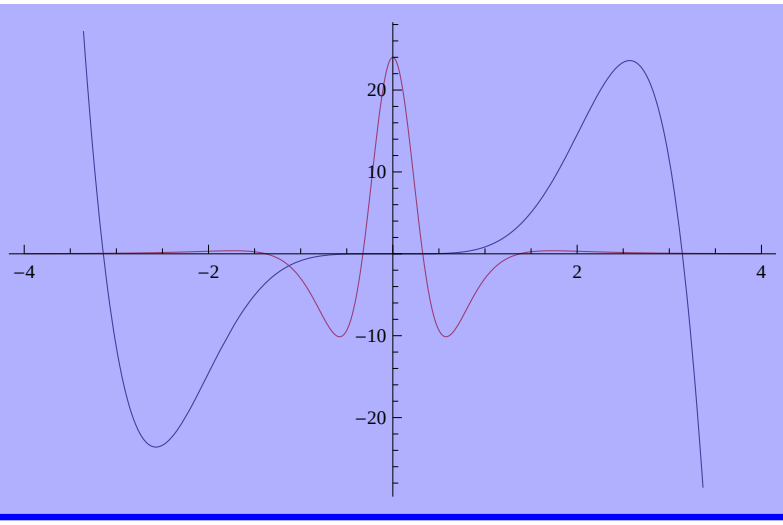
In[168]:=
```
s =.
```

In[169]:=
```
lf = LaplaceTransform[t^4 Sin[t], t, s]
```

Out[169]=
$$\frac{24\left(1-10\,s^2+5\,s^4\right)}{\left(1+s^2\right)^5}$$

In[170]:=
```
Plot[{t^4 Sin[t], lf /. s → t}, {t, -4, 4}]
```

Out[170]=

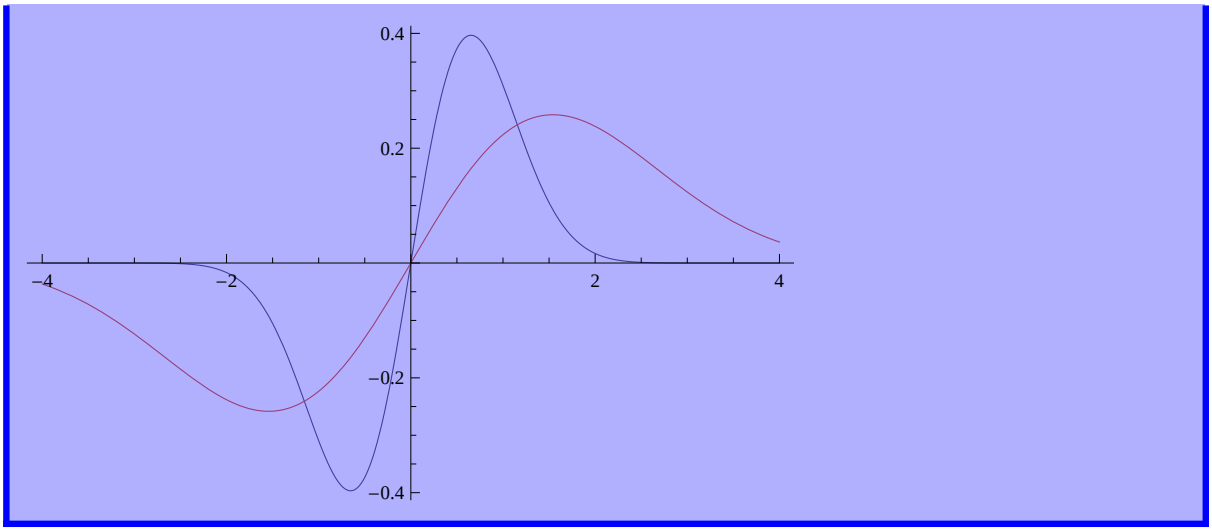


In[171]:=
```
ft = FourierTransform[Exp[-t^2] Sin[t], t, ω]
```

Out[171]=
$$\frac{1}{2\sqrt{2}}\,i\,(-1+\text{Cosh}[\omega]+\text{Sinh}[\omega])\left(\text{Cosh}\left[\frac{1}{4}(1+\omega)^2\right]-\text{Sinh}\left[\frac{1}{4}(1+\omega)^2\right]\right)$$

In[172]:=
```
Plot[{Exp[-t^2] Sin[t], Im[ft] /. ω → t}, {t, -4, 4}]
```

Out[172]=



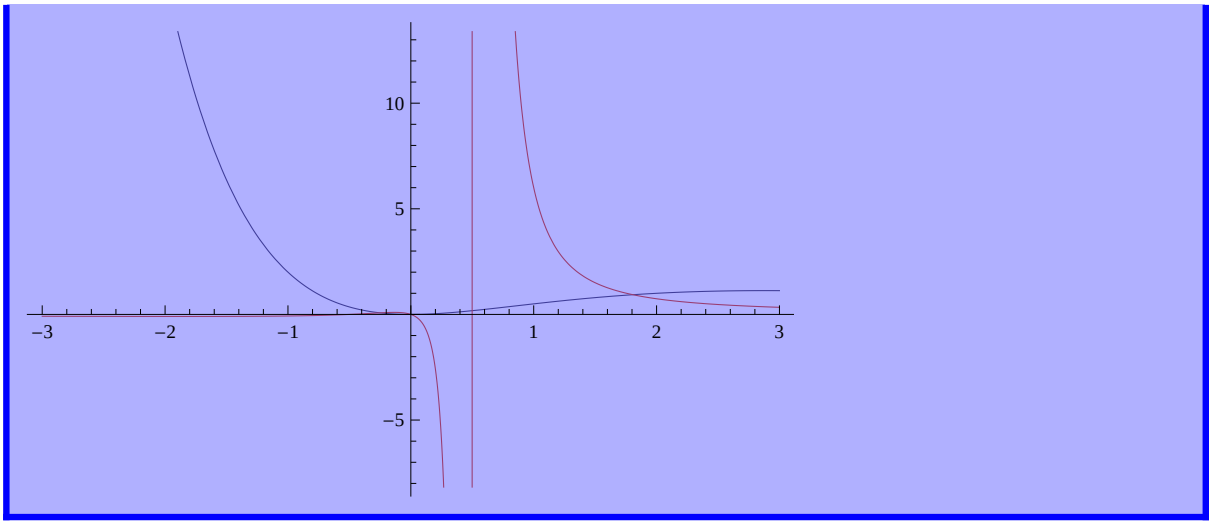
In[173]:=
```
zf = ZTransform[n^2 2^(-n), n, z]
```

Out[173]=

$$\frac{2\,z\,(1+2\,z)}{(-1+2\,z)^3}$$

In[174]:=
```
Plot[{n^2 2^(-n), zf /. z → n}, {n, -3, 3}]
```

Out[174]=



■ Szeregi

Series[funkcja,{zmienna,x0,stopien}]
Normal[szereg]

In[175]:=
```
f1 = Series[Sin[x], {x, 0, 5}]
```

Out[175]=
$$x - \frac{x^3}{6} + \frac{x^5}{120} + O[x]^6$$

In[176]:=
```
Normal[f1]
```

Out[176]=
$$x - \frac{x^3}{6} + \frac{x^5}{120}$$

In[177]:=
```
f2 = Series[Sin[x], {x, Pi / 2, 5}]
```

Out[177]=
$$1 - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{24}\left(x - \frac{\pi}{2}\right)^4 + O\left[x - \frac{\pi}{2}\right]^6$$

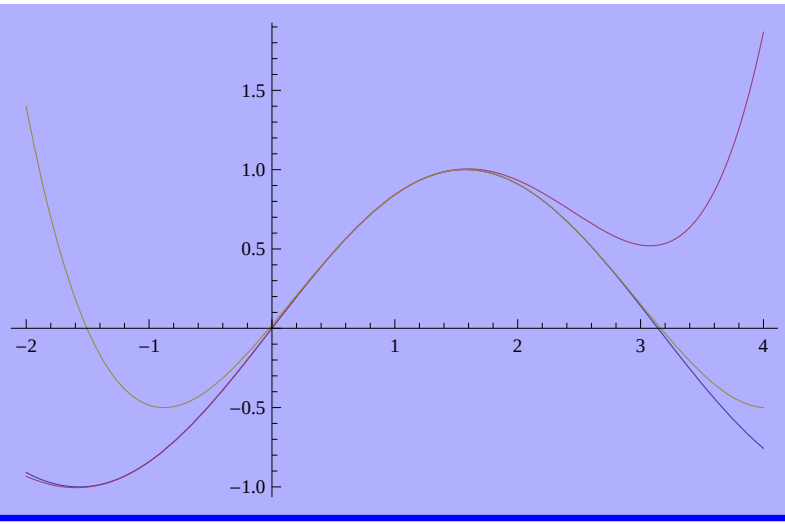
In[178]:=
```
f1 = Normal[f1]
f2 = Normal[f2]
```

Out[178]=
$$x - \frac{x^3}{6} + \frac{x^5}{120}$$

Out[179]=
$$1 - \frac{1}{2}\left(-\frac{\pi}{2} + x\right)^2 + \frac{1}{24}\left(-\frac{\pi}{2} + x\right)^4$$

In[180]:=
```
Plot[{Sin[x], f1, f2}, {x, -2, 4}]
```

Out[180]=



■ Rownania rozniczkowe

DSolve[rownanie, funkcja, zmienna]
DSolve[{rown1,rown2,...},{f1, f2,...},zmienna]

In[181]:=
```
DSolve[y'[x] == Sin[x], y[x], x]
```

Out[181]=
```
{{y[x] → C[1] - Cos[x]}}
```

In[182]:=
```
DSolve[{y'[x] == Sin[x], y[0] == 2}, y[x], x]
```

Out[182]=
```
{{y[x] → 3 - Cos[x]}}
```

In[183]:=
```
DSolve[y'[x] + 2 y[x] == Cos[x], y[x], x]
```

Out[183]=

$$\left\{\left\{y[x] \to e^{-2x} C[1] + \frac{1}{5}\left(2\,\text{Cos}[x] + \text{Sin}[x]\right)\right\}\right\}$$

■ Systemy liczenia

BaseForm[liczba, podstawa]
podstawa^^liczba

In[185]:=
```
BaseForm[103 422, 16]
```

Out[185]//BaseForm=
$193fe_{16}$

In[186]:=
```
BaseForm[103 422, 37]
```

BaseForm::basf : Requested base 37 should be an integer between 2 and 36. ≫

Out[186]//BaseForm=
```
BaseForm[103 422, 37]
```

In[187]:=
```
9^^1010
```

Out[187]=
```
738
```